UNIVERSITY OF TIKRIT
ENGINEERING COLLEGE
Chemical & Mechanical Engineering Department

Engineering Mechanics
Statics Lectures

Chapter three

Lecturer
Sabah Mahdi salih

2018 -2019

Chapter three

3-1 EQUILIBRIUM

A body will be in equilibrium when the resultant of *all* forces acting on it is zero. And the resultant **M** or couple **M** are zero, So the equilibrium equations will be:

$$\mathbf{R} = \Sigma \mathbf{F} = 0 \qquad \mathbf{M} = \Sigma \mathbf{M} = 0$$

يكون الجسم في حالة توازن في احد الحالتين وهما :
اولا: عندما يكون الجسم في حالة السكون (عدم وجود قوى خارجية مؤثرة - Rest state)
ثانيا: عندما تؤثر على الجسم عدة قوى بشرط ان تكون محصلة القوى (مجموع تاثير هذه القوى) في كل اتجاه يساوي صفر ويكون مجموع العزوم حول اي نقطة في ذلك الجسم مساوي الى الصفر.

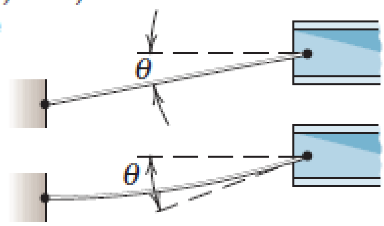
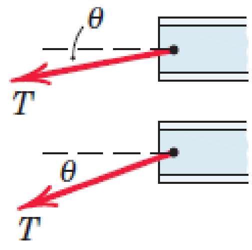
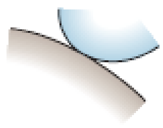
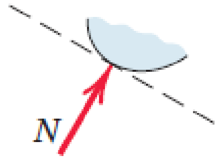
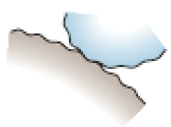
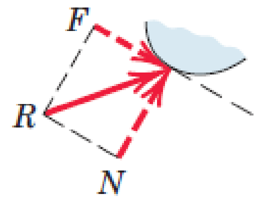
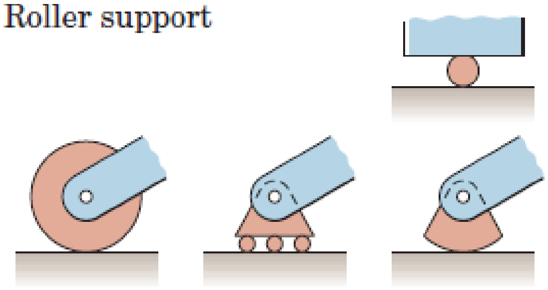
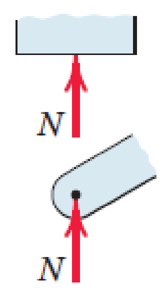
**

3-1-1 SYSTEM ISOLATION AND THE FREE-BODY DIAGRAM:

A *mechanical system* is defined as a body or group of bodies which can be conceptually isolated from all other bodies. A system may be a single body or a combination of connected bodies. The bodies may be rigid or nonrigid. The system may also be an identifiable fluid mass, either liquid or gas, or a combination of fluids and solids. In statics we study primarily forces which act on rigid bodies at rest, although we also study forces acting on fluids in equilibrium.

Modeling the Action of Forces

Figure 3/1 shows the common types of force application on mechanical systems for analysis in two dimensions. Each example shows the force exerted *on* the body to be *isolated*, *by* the body to be *removed*. Newton’s third law, which notes the existence of an equal and opposite reaction to every action, must be carefully observed. The force exerted *on* the body in question *by* a contacting or supporting member is always in the sense to oppose the movement of the isolated body which would occur if the contacting or supporting body were removed.

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent *F* (frictional force) as well as a normal component *N* of the resultant
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

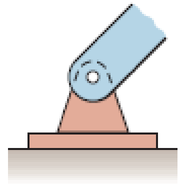
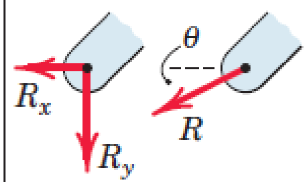
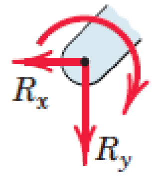
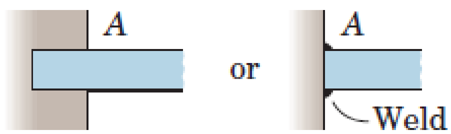
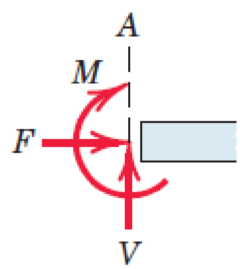
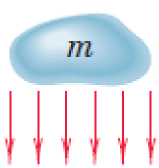
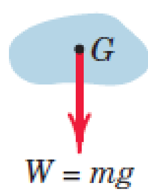
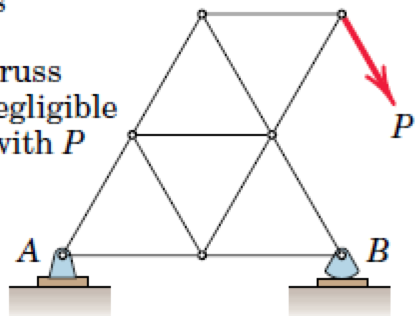
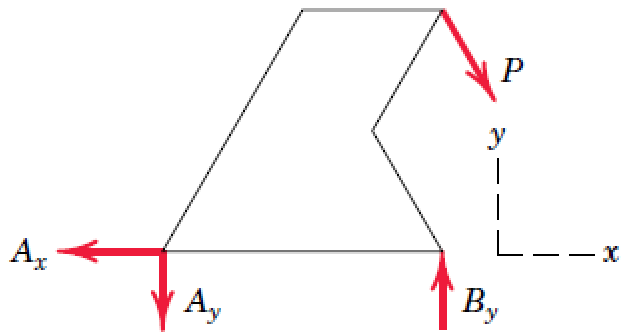
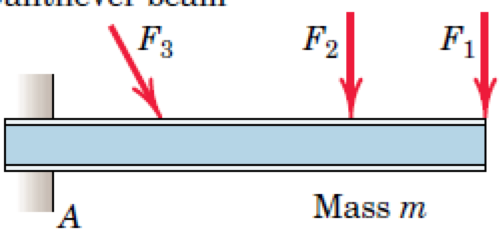
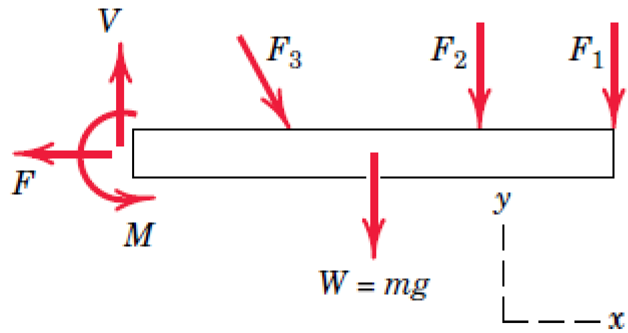
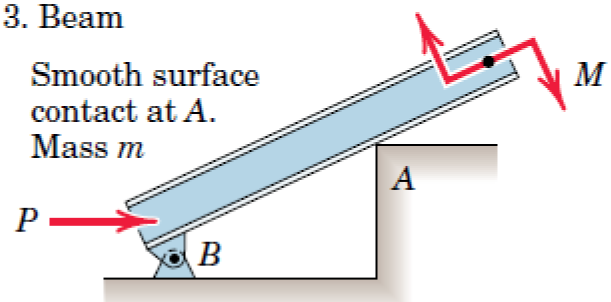
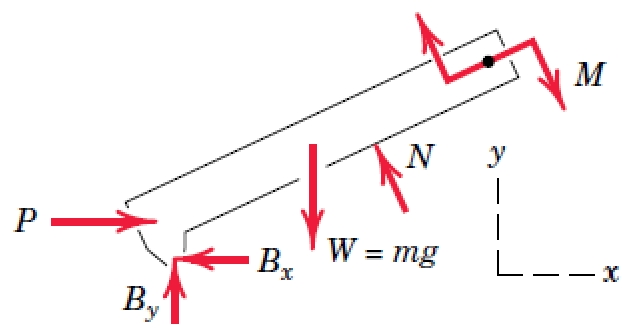
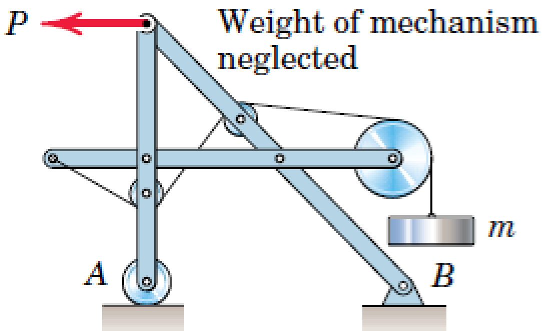
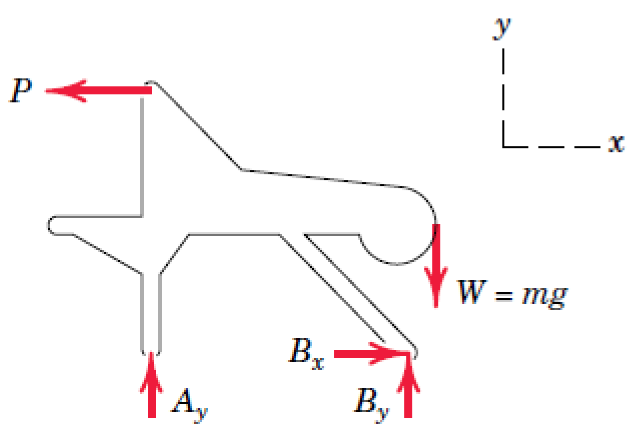
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)		
Type of Contact and Force Origin	**Action on Body to Be Isolated**	
6. Pin connection 	Pin free to turn  Pin not free to turn 	A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 		The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.

Fig.(3)

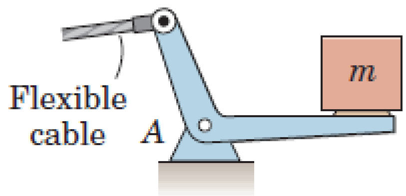
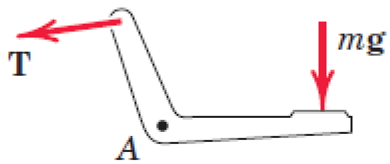
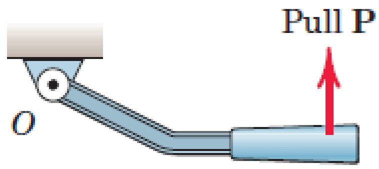
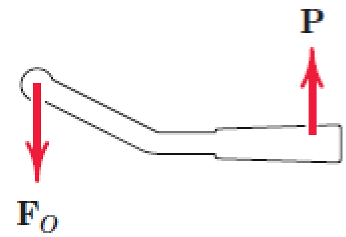
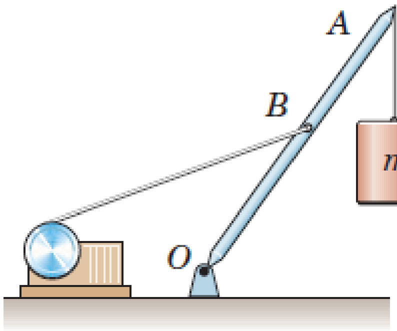
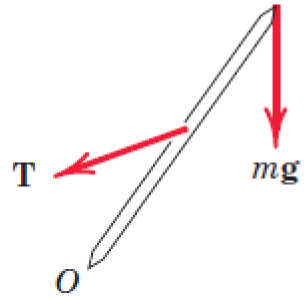
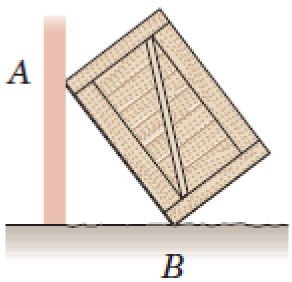
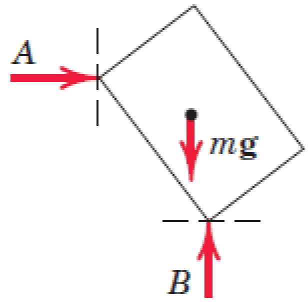
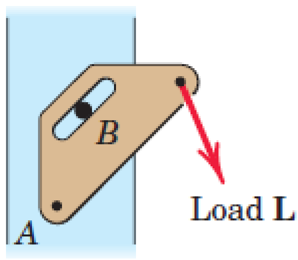
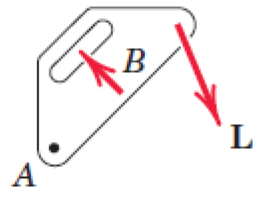
Construction of Free-Body Diagrams

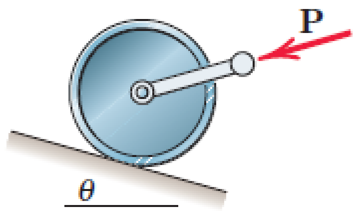
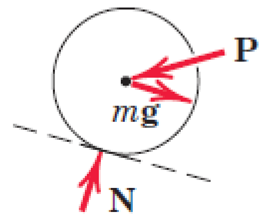
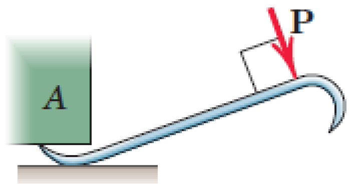
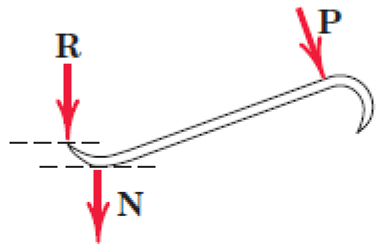
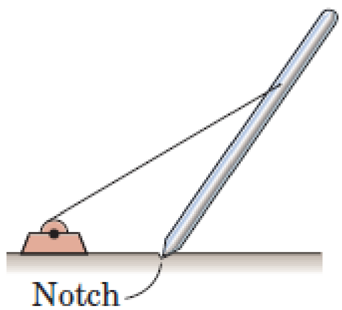
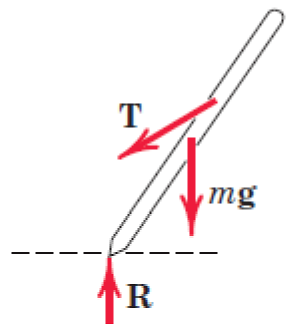
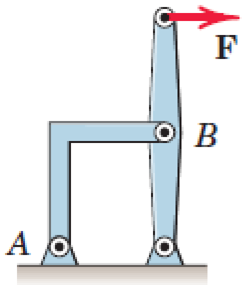
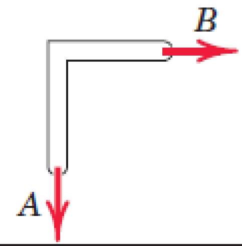
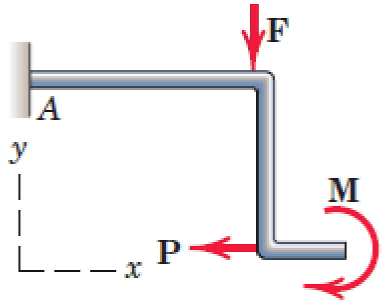
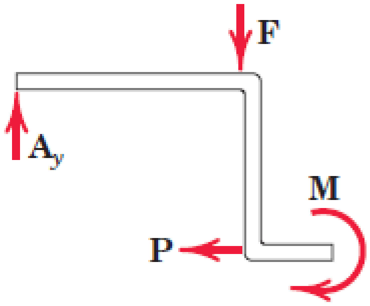
Examples of Free-Body Diagrams

Figure 3/2 gives four examples of mechanisms and structures together with their correct free-body diagrams. Dimensions and magnitudes are omitted for clarity. In each case we treat the entire system as a single body, so that the internal forces are not shown. The characteristics of the various types of contact forces illustrated in Fig. 3/1 are used in the four examples as they apply.

SAMPLE FREE-BODY DIAGRAMS	
Mechanical System	Free-Body Diagram of Isolated Body
1. Plane truss Weight of truss assumed negligible compared with P 	
2. Cantilever beam 	
3. Beam Smooth surface contact at A. Mass m 	
4. Rigid system of interconnected bodies analyzed as a single unit Weight of mechanism neglected 	

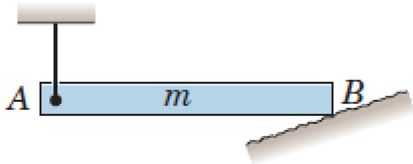
FREE-BODY DIAGRAM EXERCISES

	Body	Incomplete FBD
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

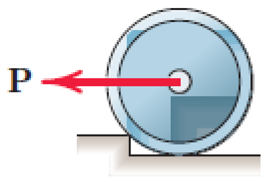
	Body	Wrong or Incomplete FBD
1. Lawn roller of mass m being pushed up incline θ .		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		

Draw a complete and correct free-body diagram of each of the bodies designated in the statements. The weights of the bodies are significant only if the mass is stated.

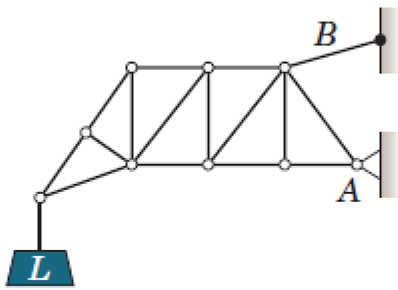
1. Uniform horizontal bar of mass m suspended by vertical cable at A and supported by rough inclined surface at B.



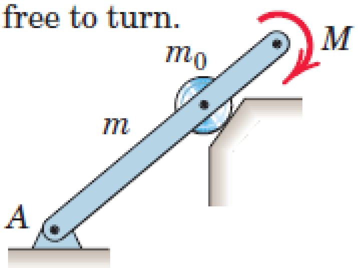
2. Wheel of mass m on verge of being rolled over curb by pull **P**.



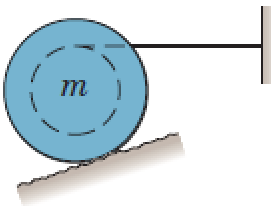
3. Loaded truss supported by pin joint at A and by cable at B.



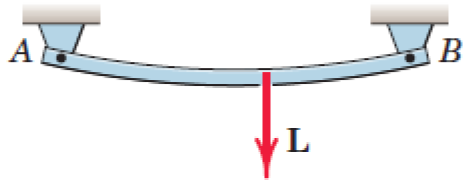
4. Uniform bar of mass m and roller of mass m_0 taken together. Subjected to couple M and supported as shown. Roller is free to turn.



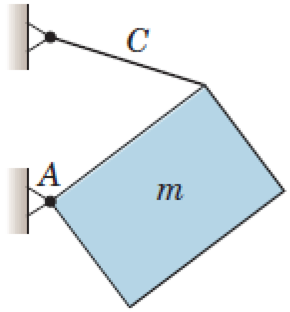
5. Uniform grooved wheel of mass m supported by a rough surface and by action of horizontal cable.



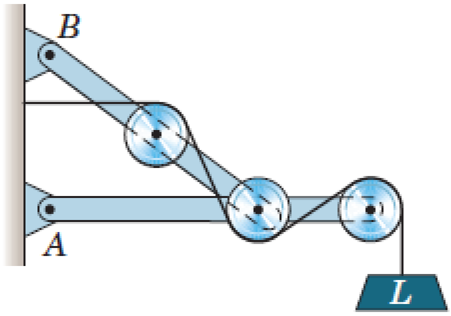
6. Bar, initially horizontal but deflected under load **L**. Pinned to rigid support at each end.



7. Uniform heavy plate of mass m supported in vertical plane by cable C and hinge A.

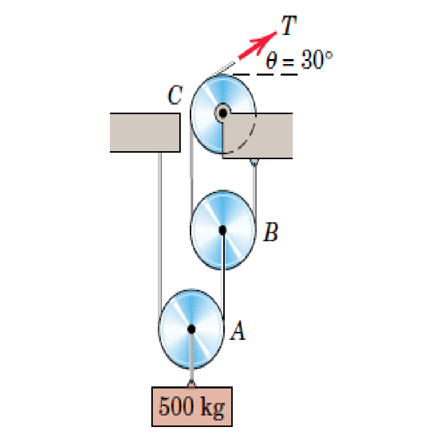


8. Entire frame, pulleys, and contacting cable to be isolated as a single unit.



Sample Problem 3/2

Calculate the tension T in the cable which supports the 500-kg mass with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.



Solution. The free-body diagram of each pulley is drawn in its relative position to the others. We begin with pulley A, which includes the only known force. With the unspecified pulley radius designated by r, the equilibrium of moments about its center O and the equilibrium of forces in the vertical direction require

$[\Sigma M_O = 0] \qquad T_1 r - T_2 r = 0 \qquad T_1 = T_2$

$[\Sigma F_y = 0] \qquad T_1 + T_2 - 500(9.81) = 0 \qquad 2T_1 = 500(9.81) \qquad T_1 = T_2 = 2450 \text{ N}$

From the example of pulley A we may write the equilibrium of forces on pulley B by inspection as

$$T_3 = T_4 = T_2/2 = 1226 \text{ N}$$

For pulley C the angle $\theta = 30°$ in no way affects the moment of T about the center of the pulley, so that moment equilibrium requires

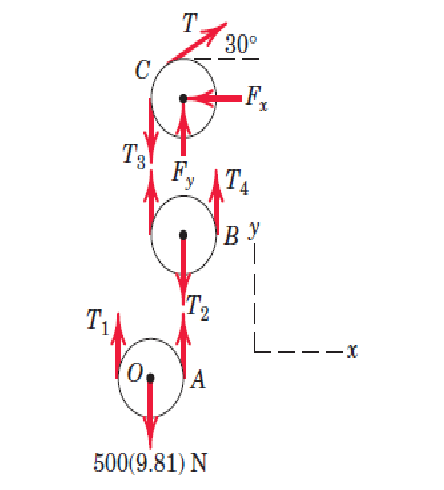
$$T = T_3 \qquad \text{or} \qquad T = 1226 \text{ N} \qquad \textit{Ans.}$$

Equilibrium of the pulley in the x- and y-directions requires

$[\Sigma F_x = 0] \qquad 1226 \cos 30° - F_x = 0 \qquad F_x = 1062 \text{ N}$

$[\Sigma F_y = 0] \qquad F_y + 1226 \sin 30° - 1226 = 0 \qquad F_y = 613 \text{ N}$

$[F = \sqrt{F_x^2 + F_y^2}] \qquad F = \sqrt{(1062)^2 + (613)^2} = 1226 \text{ N} \qquad \textit{Ans.}$

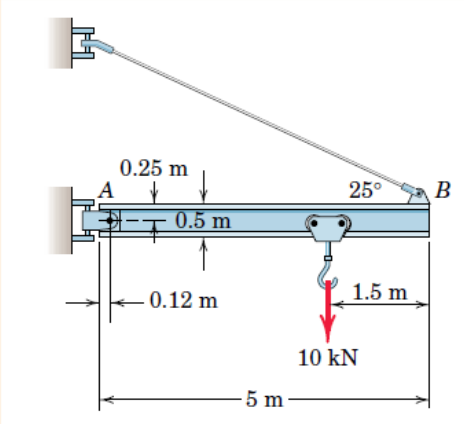


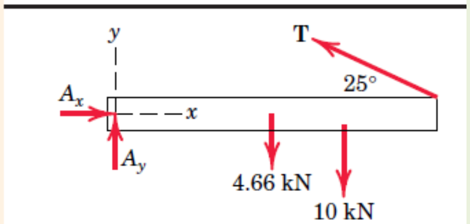
Helpful Hint

① Clearly the radius r does not influence the results. Once we have analyzed a simple pulley, the results should be perfectly clear by inspection.

Sample Problem 3/4

Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.





Free-body diagram

Algebraic solution. The system is symmetrical about the vertical x-y plane through the center of the beam, so the problem may be analyzed as the equilibrium of a coplanar force system. The free-body diagram of the beam is shown in the figure with the pin reaction at A represented in terms of its two rectangular components. The weight of the beam is $95(10^{-3})(5)9.81 = 4.66$ kN and acts through its center. Note that there are three unknowns A_x, A_y, and T, which may be found from the three equations of equilibrium. We begin with a moment equation about A, which eliminates two of the three unknowns from the equation. In applying the moment equation about A, it is simpler to consider the moments of the x- and y-components of $\mathbf{T}$ than it is to compute the perpendicular distance from $\mathbf{T}$ to A. Hence, with the counterclockwise sense as positive we write

$$[\Sigma M_A = 0] \qquad (T \cos 25°)0.25 + (T \sin 25°)(5 - 0.12) - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$$

from which $\qquad T = 19.61 \text{ kN} \qquad$ *Ans.*

Equating the sums of forces in the x- and y-directions to zero gives

$$[\Sigma F_x = 0] \qquad A_x - 19.61 \cos 25° = 0 \qquad A_x = 17.77 \text{ kN}$$

$$[\Sigma F_y = 0] \qquad A_y + 19.61 \sin 25° - 4.66 - 10 = 0 \qquad A_y = 6.37 \text{ kN}$$

$$\left[A = \sqrt{A_x^2 + A_y^2}\right] \qquad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88 \text{ kN} \qquad \textit{Ans.}$$

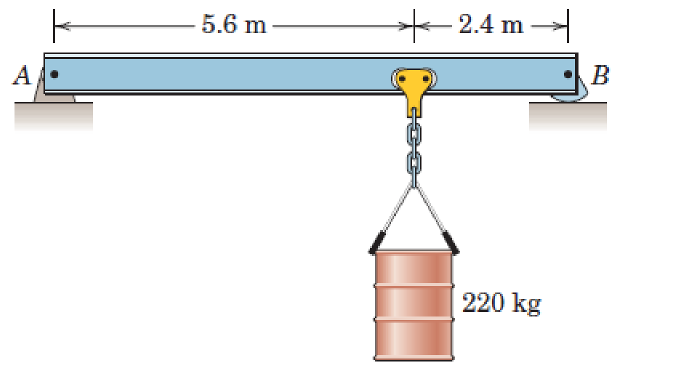
Helpful Hints

① The justification for this step is Varignon's theorem, explained in Art. 2/4. Be prepared to take full advantage of this principle frequently.

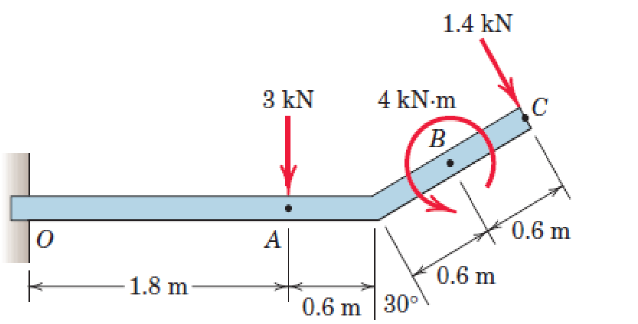
② The calculation of moments in two-dimensional problems is generally handled more simply by scalar algebra than by the vector cross product $\mathbf{r} \times \mathbf{F}$. In three dimensions, as we will

Prob: (1) (2)

(1) The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.



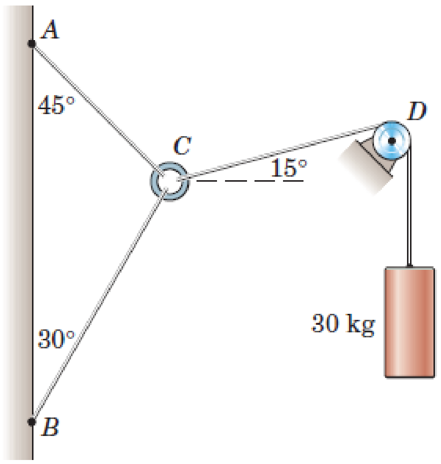
(2) **7** The uniform beam has a mass of 50 kg per meter of length. Compute the reactions at the support O. The force loads shown lie in a vertical plane.
Ans. $O_x = -0.7$ kN, $O_y = 5.98$ kN, $M_O = 9.12$ kN·m



(3)

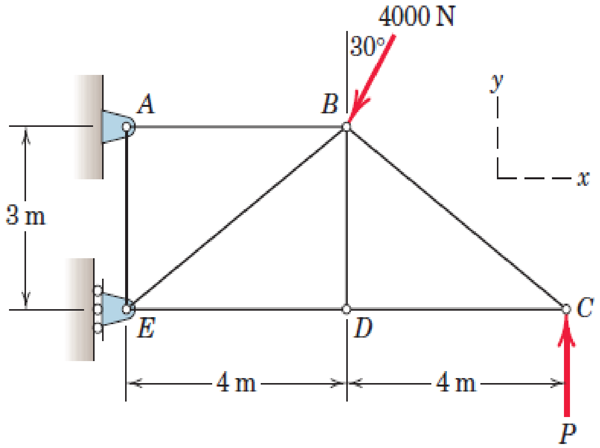
Three cables are joined at the junction ring C. Determine the tensions in cables AC and BC caused by the weight of the 30-kg cylinder.

Ans. $T_{AC} = 215$ N, $T_{BC} = 264$ N



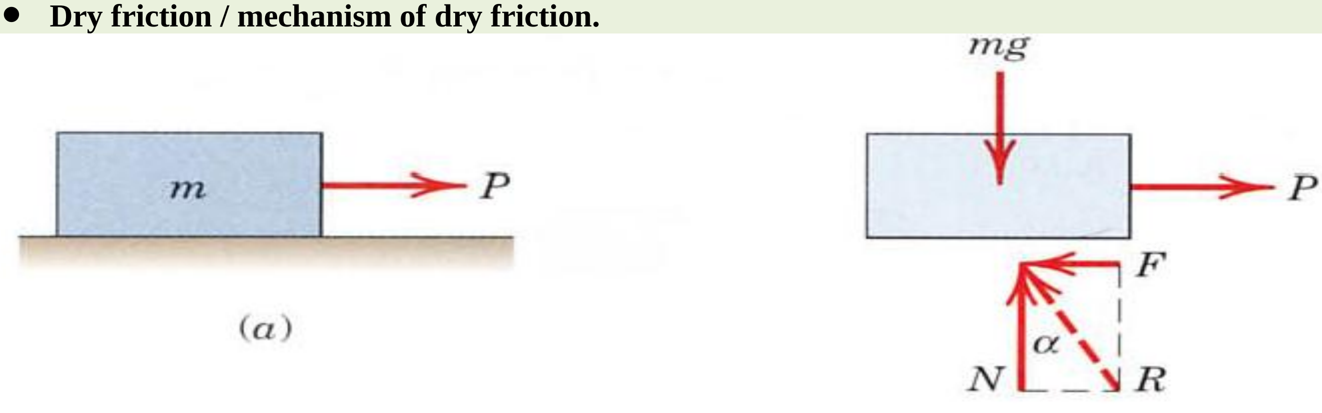
(4)

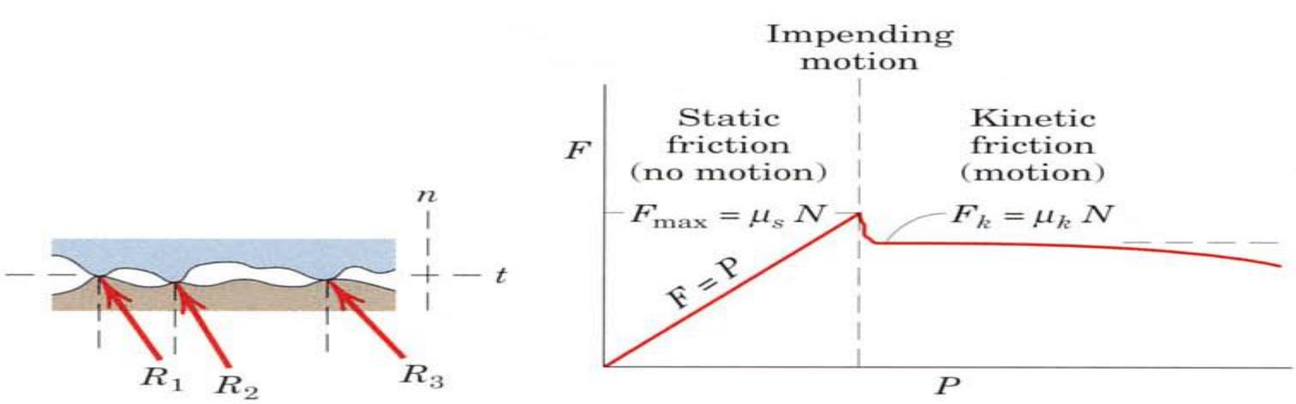
0 Determine the reactions at A and E if $P = 500$ N. What is the maximum value which P may have for static equilibrium? Neglect the weight of the structure compared with the applied loads.



3-2 The Friction

- **Dry friction / mechanism of dry friction.**





Static Friction

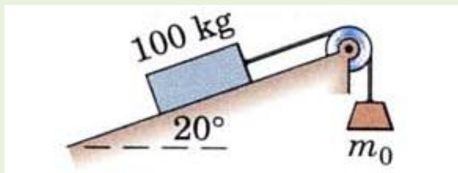
The region in Fig. 6/1*d* up to the point of slippage or impending motion is called the range of *static friction*, and in this range the value of the friction force is determined by the *equations of equilibrium*. This friction force may have any value from zero up to and including the maximum value. For a given pair of mating surfaces the experiment shows that this maximum value of static friction F_{max} is proportional to the normal force N. Thus, we may write

$$\boxed{F_{max} = \mu_s N} \qquad \textbf{(6/1)}$$

Ex:1

Sample Problem 6/2

Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.



Solution. The maximum value of m_0 will be given by the requirement for motion impending up the plane. The friction force on the block therefore acts down the plane, as shown in the free-body diagram of the block for Case I in the figure. With the weight $mg = 100(9.81) = 981$ N, the equations of equilibrium give

$[\Sigma F_y = 0]$ $\quad N - 981 \cos 20° = 0 \quad N = 922$ N

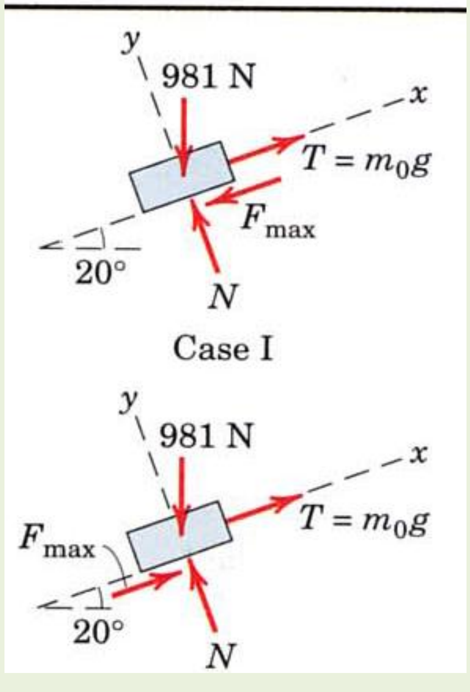
$[F_{max} = \mu_s N]$ $\quad F_{max} = 0.30(922) = 277$ N

$[\Sigma F_x = 0]$ $\quad m_0(9.81) - 277 - 981 \sin 20° = 0 \quad m_0 = 62.4$ kg $\qquad$ *Ans.*

The minimum value of m_0 is determined when motion is impending down the plane. The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free-body diagram for Case II. Equilibrium in the *x*-direction requires

$[\Sigma F_x = 0]$ $\quad m_0(9.81) + 277 - 981 \sin 20° = 0 \quad m_0 = 6.01$ kg $\qquad$ *Ans.*

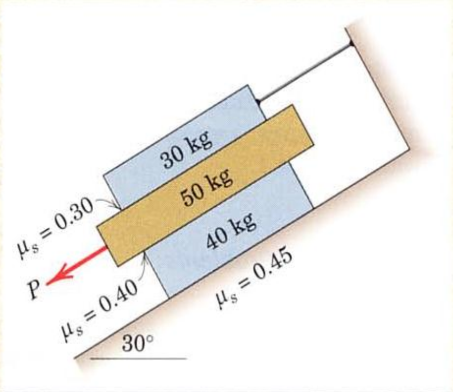
Thus, m_0 may have any value from 6.01 to 62.4 kg, and the block will remain at rest.



Ex:2

Sample Problem 6/5

The three flat blocks are positioned on the 30° incline as shown, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown. Determine the maximum value which P may have before any slipping takes place.



Solution. The free-body diagram of each block is drawn. The friction forces are assigned in the directions to oppose the relative motion which would occur if no friction were present. There are two possible conditions for impending motion. Either the 50-kg block slips and the 40-kg block remains in place, or the 50- and 40-kg blocks move together with slipping occurring between the 40-kg block and the incline.

The normal forces, which are in the y-direction, may be determined without reference to the friction forces, which are all in the x-direction. Thus,

$[\Sigma F_y = 0]$ (30-kg) $N_1 - 30(9.81)\cos 30° = 0$ $\quad N_1 = 255$ N

(50-kg) $N_2 - 50(9.81)\cos 30° - 255 = 0$ $\quad N_2 = 680$ N

(40-kg) $N_3 - 40(9.81)\cos 30° - 680 = 0$ $\quad N_3 = 1019$ N

We will assume arbitrarily that only the 50-kg block slips, so that the 40-kg block remains in place. Thus, for impending slippage at both surfaces of the 50-kg block, we have

$[F_{max} = \mu_s N]$ $\quad F_1 = 0.30(255) = 76.5$ N $\quad F_2 = 0.40(680) = 272$ N

The assumed equilibrium of forces at impending motion for the 50-kg block gives

$[\Sigma F_x = 0]$ $\quad P - 76.5 - 272 + 50(9.81)\sin 30° = 0$ $\quad P = 103.1$ N

We now check on the validity of our initial assumption. For the 40-kg block with $F_2 = 272$ N the friction force F_3 would be given by

$[\Sigma F_x = 0]$ $\quad 272 + 40(9.81)\sin 30° - F_3 = 0$ $\quad F_3 = 468$ N

But the maximum possible value of F_3 is $F_3 = \mu_s N_3 = 0.45(1019) = 459$ N. Thus, 468 N cannot be supported and our initial assumption was wrong. We conclude, therefore, that slipping occurs first between the 40-kg block and the incline. With the corrected value $F_3 = 459$ N, equilibrium of the 40-kg block for its impending motion requires

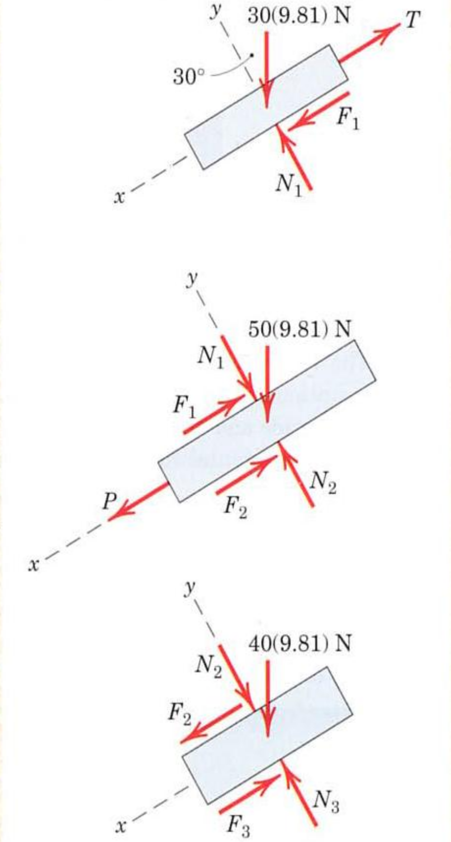
$[\Sigma F_x = 0]$ $\quad F_2 + 40(9.81)\sin 30° - 459 = 0$ $\quad F_2 = 263$ N

Equilibrium of the 50-kg block gives, finally,

$[\Sigma F_x = 0]$ $\quad P + 50(9.81)\sin 30° - 263 - 76.5 = 0$

$P = 93.8$ N *Ans.*

Thus, with $P = 93.8$ N, motion impends for the 50-kg and 40-kg blocks as a unit.



Helpful Hints

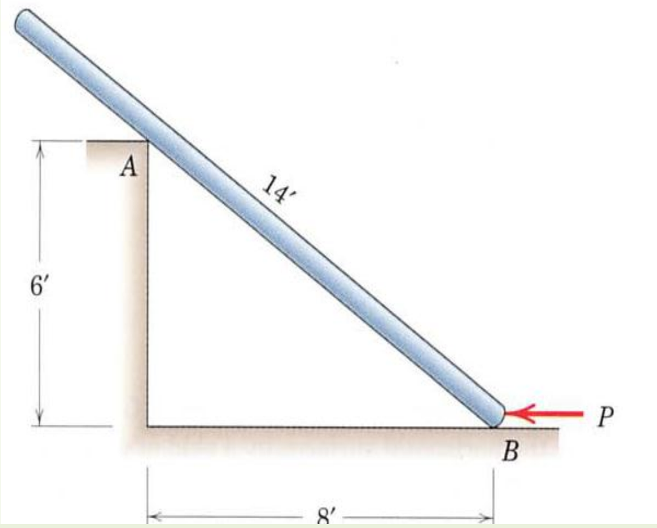
① In the absence of friction the middle block, under the influence of P, would have a greater movement than the 40-kg block, and the friction force F_2 will be in the direction to oppose this motion as shown.

② We see now that F_2 is less than $\mu_s N_2 = 272$ N.

Problems.

(1)

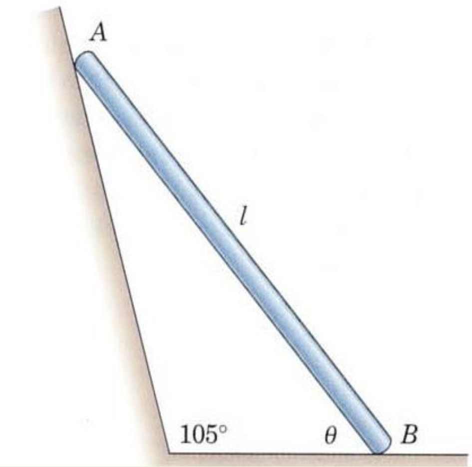
12 The uniform 14-ft pole weighs 150 lb and is supported as shown. Calculate the force P required to move the pole if the coefficient of static friction for each contact location is 0.40.



(2)

The uniform pole of length l and mass m is placed against the supporting surfaces shown. If the coefficient of static friction is $\mu_s = 0.25$ at both A and B, determine the maximum angle θ at which the pole can be placed before it begins to slip.

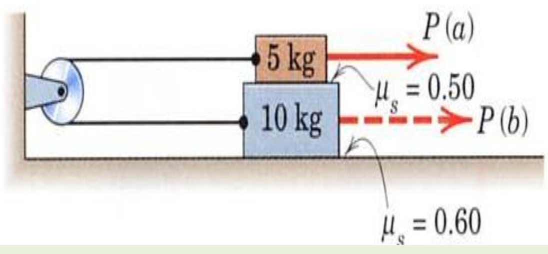
Ans. $\theta = 59.9°$



(3)

The system of two blocks, cable, and fixed pulley is initially at rest. Determine the horizontal force P necessary to cause motion when (a) P is applied to the 5-kg block and (b) P is applied to the 10-kg block. Determine the corresponding tension T in the cable for each case.

Ans. (a) P = 137.3 N, T = 112.8 N
(b) P = 137.3 N, T = 24.5 N



(4)

Determine the distance s to which the 90-kg painter can climb without causing the 4-m ladder to slip at its lower end A. The top of the 15-kg ladder has a small roller, and at the ground the coefficient of static friction is 0.25. The mass center of the painter is directly above her feet.

Ans. s = 2.55 m